CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

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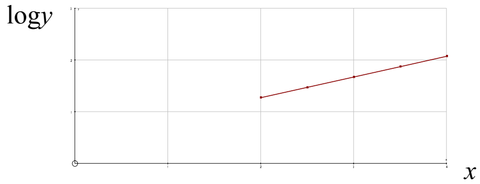
CAMBRIDGE
International Examinations

1	$\frac{dy}{dx} = 2x - \frac{16}{x^2}$ When $\frac{dy}{dx} = 0$, $x = 2,\ y = 12$	**M1** **A1** **DM1** **A1**	for attempt to differentiate all correct for equating $\frac{dy}{dx}$ to zero and an attempt to solve for x. **A1** for both, but no extra solutions
2 (a)		**B1** **B1** **B1**	for correct shape for max value of 2, starting at (0, 2) and finishing at (180°, 2) for min value of –4
(b) (i)	4	**B1**	must be positive
(ii)	60° or $\frac{\pi}{3}$ or 1.05 rad	**B1**	
3 (i)	$y = 4(x+3)^{\frac{1}{2}}(+c)$ $10 = 4\left(9^{\frac{1}{2}}\right) + c$ $c = -2$ $y = 4(x+3)^{\frac{1}{2}} - 2$	**M1, A1** **M1** **A1**	**M1** for $(x+3)^{\frac{1}{2}}$, **A1** for $4(x+3)^{\frac{1}{2}}$ for a correct attempt to find *c*, but must be from an attempt to integrate Allow **A1** for $c = -2$
(ii)	$6 = 4(x+3)^{\frac{1}{2}} - 2$ $x = 1$	**A1 ft**	**ft** for substitution into *their* equation to obtain x; must have the first **M1**

4 **(i)**	$5y^2 - 7y + 2 = 0$	**B1, B1**	**B1** for 5, **B1** for –7
(ii)	$(5y-2)(y-1)=0$	**M1**	for solution of quadratic equation from **(i)**
	$y = \frac{2}{5}, x = \frac{\ln 0.4}{\ln 5}$	**M1**	for use of logarithms to solve equation of the type $5^x = k$
	$x = -0.569$	**A1**	must be evaluated to 3sf or better
	$y = 1, x = 0$	**B1**	
5 **(i)**	$\frac{dy}{dx} = 3x^2 - \frac{1}{x}$	**M1**	for attempt to differentiate
	When $x = 1$, $y = 1$ and $\frac{dy}{dx} = 2$	**B1**	for $y = 1$
	Tangent: $y - 1 = 2(x-1)$	**DM1**	for attempt to find equation of tangent
	$(y = 2x - 1)$	**A1**	allow equation unsimplified
(ii)	Mid-point (5, 9)	**B1**	for midpoint from given coordinates
	$9 = 2(5) - 1$	**B1**	for checking the mid-point lies on tangent
	Alternative Method: Tangent equation $y = 2x - 1$ Equation of line joining (–2, 16) and (12, 2) $y = -x + 14$		
	Solve simultaneously $x = 5$, $y = 9$	**B1**	for a complete method to find the coordinates of the point of intersection
	Mid-point (5, 9)	**B1**	for midpoint from given coordinates
6 **(i)**	$(2+px)^6 = 64 + 192px + 240p^2x^2 \ldots$	**B1**	for $240p^2$ or $240p^2x^2$ or $^6C_2 \times 2^4 \times (px)^2$ or $^6C_2 \times 2^4 \times p^2$ or $^6C_2 \times 2^4 \times p^2x^2$
	$240p^2 = 60$	**M1**	for equating *their* term in x^2 to 60 and attempt to solve
	$p = \frac{1}{2}$	**A1**	
(ii)	$(3-x)(64 + 192px + 240p^2x^2 \ldots)$	**B1 ft**	**ft** for $192p$, 96 or $192 \times$ *their* p
	Coefficient of x^2 is $180 - 192p$ = 84	**M1** **A1**	for $180 - 192p$

7 **(i)**	$\mathbf{A}^{-1} = \frac{1}{5ab}\begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$	**B1, B1**	**B1** for $\frac{1}{5ab}$, **B1** for $\begin{pmatrix} b & -2b \\ a & 3a \end{pmatrix}$
(ii)	$\mathbf{X} = \mathbf{BA}^{-1}$	**M1**	for post-multiplication by inverse matrix
	$= \begin{pmatrix} -a & b \\ 2a & 2b \end{pmatrix}\begin{pmatrix} \frac{1}{5a} & -\frac{2}{5a} \\ \frac{1}{5b} & \frac{3}{5b} \end{pmatrix}$	**DM1**	for correct attempt at matrix multiplication, needs at least one term correct for their BA^{-1} (allow unsimplified)
	$= \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix}$	**A1** **A1**	for each correct pair of elements, must be simplified
8 **(i)**	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$, at P, $x = -2 + \frac{1}{4}(12)$ so at P, $x = 1$	**B1**	for convincing argument for $x = 1$
	$y = 3 + \frac{1}{4}(16)$, $y = 7$	**B1**	for $y = 7$
(ii)	Gradient of $AB = \frac{16}{12}$, so perp gradient = $-\frac{3}{4}$	**M1**	for finding gradient of perpendicular
	Perp line: $y - 7 = -\frac{3}{4}(x-1)$	**M1**	for equation of perpendicular through their P
	$(3x + 4y = 31)$	**A1**	Allow unsimplified
(iii)	$Q\left(0, \frac{31}{4}\right)$	**B1 ft**	**ft** on their perpendicular line, may be implied
		M1	for any valid method of finding the area of the correct triangle, allow use of *their* Q; must be in the form $(0, q)$.
	Area AQB = 12.5	**A1**	

<table>
<tr><td>9 (i)</td><td>$\log y = \log a + x\log b$
<table>
<tr><td>x</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr>
<tr><td>lg y</td><td>1.27</td><td>1.47</td><td>1.67</td><td>1.87</td><td>2.07</td></tr>
</table>
<table>
<tr><td></td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr>
<tr><td>ln$y$</td><td>2.93</td><td>3.39</td><td>3.84</td><td>4.31</td><td>4.76</td></tr>
</table>

</td><td>B1

M1

A2,1,0</td><td>for the statement, may be seen or implied in later work,

for attempt to draw graph of x against log y

–1 each error in points plotted</td></tr>
<tr><td>(ii)</td><td>Gradient = log b
$\lg b = 0.4$ or $\ln b = 0.92$

$b = 2.5$ (allow 2.4 to 2.6)

Intercept = log a
$\lg a = 0.47$ or $\ln a = 1.10$

$a = 3$ (allow 2.8 to 3.2)

Alternative method:
Simultaneous equations may be used provided points that are on the plotted straight line are used.

$a = 3$ (allow 2.8 to 3.2)
$b = 2.5$ (allow 2.4 to 2.6)</td><td>DM1

A1

DM1

A1

DM1

DM1

A1
A1</td><td>for attempt to find gradient and equate it to log b, dependent on M1 in (i)

for attempt to equate y-intercept to log a or use their equation with their gradient and a point on the line, dependent on M1 in (i)

for a pair of equations using points on the line, dependent on M1 in (i)
for solution of these equations, dependent on M1 in (i)

A1 for each</td></tr>
</table>

10 (a) (i)	360	**B1**	
(ii)	60	**B1**	
(iii)	36	**B1**	
(b) (i)	${}^{8}C_5 \times {}^{12}C_5$	**B1, B1**	**B1** for each, allow unevaluated with no extra terms
	$56 \times 792 = 44352$	**B1**	Final answer must be evaluated and from multiplication
(ii)	4 places are accounted for Gender no longer 'important'	**M1**	for realising that 4 places are accounted or that gender is no longer important
	Need ${}^{16}C_6 = 8008$	**A1**	for 8008
	Alternative Method $\left({}^{6}C_6 \times {}^{10}C_0\right) + \left({}^{6}C_5 \times {}^{10}C_1\right) \ldots \left({}^{6}C_0 \times {}^{10}C_6\right)$	**M1**	for at least 5 of the 7 cases, allow unsimplified
	$1 + 60 + 675 + 2400 + 3150 + 1512 + 210 = 8008$	**A1**	
11 (a)	$2\cos 3x - \dfrac{\cos 3x}{\sin 3x} = 0$ $\cos 3x\left(2 - \dfrac{1}{\sin 3x}\right) = 0$	**M1**	for use of $\cot 3x = \dfrac{\cos 3x}{\sin 3x}$, may be implied
	Leading to $\cos 3x = 0,\ 3x = 90^\circ,\ 270^\circ$	**DM1**	for attempt to solve $\cos 3x = 0$ correctly from correct factorisation to obtain x
	$x = 30^\circ, 90^\circ$	**A1**	**A1** for both, no excess solutions in the range
	and $\sin 3x = \dfrac{1}{2},\ 3x = 30^\circ,\ 150^\circ$	**DM1**	for attempt to solve $\sin 3x = \dfrac{1}{2}$ correctly to obtain x
	$x = 10^\circ, 50^\circ$	**A1**	**A1** for both, condone excess solutions
(b)	$\cos\left(y + \dfrac{\pi}{2}\right) = -\dfrac{1}{2}$ $y + \dfrac{\pi}{2} = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$	**M1**	for dealing with sec $\left(y + \dfrac{\pi}{2}\right)$ correctly
		DM1	for correct order of operations, must not mix degrees and radians
	so $y = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ (0.524, 2.62)	**A1, A1**	

12 **(i)**	$\overrightarrow{AQ} = \lambda\mathbf{b} - \mathbf{a}$	**B1**	
(ii)	$\overrightarrow{BP} = \mu\mathbf{a} - \mathbf{b}$	**B1**	
(iii)	$\overrightarrow{OR} = \mathbf{a} + \frac{1}{3}(\lambda\mathbf{b} - \mathbf{a})$ or $\lambda\mathbf{b} - \frac{2}{3}(\lambda\mathbf{b} - \mathbf{a})$	**M1**	for $\mathbf{a} + \frac{1}{3}$ *their* **(i)**
	$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\lambda\mathbf{b}$	**A1**	Allow unsimplified
(iv)	$\overrightarrow{OR} = \mathbf{b} + \frac{7}{8}(\mu\mathbf{a} - \mathbf{b})$ or $\mu\mathbf{a} - \frac{1}{8}(\mu\mathbf{a} - \mathbf{b})$	**M1**	for $\mathbf{b} + \frac{7}{8}$ *their* **(ii)**
	$= \frac{1}{8}\mathbf{b} + \frac{7}{8}\mu\mathbf{a}$	**A1**	Allow unsimplified
(v)	$\frac{2}{3}\mathbf{a} + \frac{1}{3}\lambda\mathbf{b} = \frac{1}{8}\mathbf{b} + \frac{7}{8}\mu\mathbf{a}$	**M1**	for equating **(iii)** and **(iv)** and then equating like vectors
	$\frac{2}{3} = \frac{7}{8}\mu,\ \mu = \frac{16}{21}$ Allow 0.762	**A1**	
	$\frac{1}{3}\lambda = \frac{1}{8},\ \lambda = \frac{3}{8}$ Allow 0.375	**A1**	